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Estimating Wildland Fire Rate of Spread in a Spatially Nonuniform Environment

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ABSTRACT. Estimating rate of fire spread is a key element in planning for effective fire control. Land managers use the Rothermel spread model, but the model assumptions are violated when fuel, weather, and topography are nonuniform. This paper compares three averaging techniques—arithmetic mean of spread rates, spread based on mean fuel conditions, and harmonic mean of spread rates—used to estimate the effective rate-of-spread in heterogeneous environments. For particular ranges of the independent variables of the spread model, there is a well-defined ordering of the averages—a consequence of the convexity of the spread function. The harmonic mean of spread rates along the burn path is offered as an appropriate estimator of fire spread rate in a nonuniform field. FOREST SCI. 31:21–29.

ADDITIONAL KEY WORDS. Fire spread, harmonic mean, Jensen's inequality, surface area-to-volume ratio, fire behavior.

ESTIMATING RATE-OF-SPREAD is a key element in planning for effective wildland fire control. Since the early 1970's, land managers have estimated fire spread potential with a model that accounts for the effects of fuel, weather, and topography (Rothermel 1972). The model forms an integral part of the USDA Forest Service National Fire Danger Rating System (Deeming and others 1972; 1977). Conceptually, the model estimates the quasi-steady state linear rate-of-spread of the flaming front in an environment specified by fuel, weather, and topographic descriptors. Sneeuwjagt and Frandsen (1977), and Andrews (1980) report favorable statistical comparisons between model-predicted and observed rates-of-spread, when burning conditions are uniform.

Nonuniform fuel, weather, and topography over an area complicate rate-of-spread estimation, because the spread model assumes that the descriptors are uniform in some interval. Spatial uniformity is more often an exception than a rule, so averaging techniques have been used with the spread model to estimate the effective rate-of-spread in heterogeneous environments. Rothermel (1982) uses the arithmetic mean of the spread rates, weighted by the proportion of area covered by each of two codominant fuels, in the two-fuel-model concept. Call the arithmetic mean of spread rates an A-type estimator (hereafter *A*). Frandsen and Andrews (1979) compared *A* with the spread value computed from average fuel conditions. They constructed detailed nonuniform slash and grass-sagebrush fuel arrays in a computer to simulate fire behavior in spatially heterogeneous fuelbeds. By computing spread using the average fuel conditions, they essentially assume a uniform fuelbed, for which the corresponding rate-of-spread estimator is called the U-type (hereafter *U*), for "uniform" fuel model. Frandsen and Andrews observed that *U* tended to be smaller than *A*, which suggests that choosing one over the other commits one to higher or lower spread estimates. Which is appropriate?

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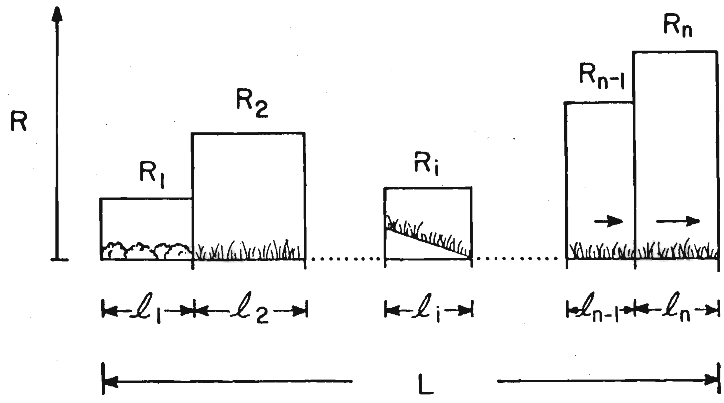

FIGURE 1. Characterization of nonuniform fire spread along a burn path of length L. Rate-of-spread (R) varies with changes in fuel type (l_1 to l_2), slope (l_i), and wind (l_{n-1} to l_n). Segment i has length l_i, and burns at the rate R_i, as computed with the Rothermel model.

This paper proposes the harmonic mean of spread rates, H, as an appropriate rate-of-spread estimator under nonuniform burning conditions. It justifies by a simple axiomatic argument the use of H—which is not much more difficult to compute than A—as a logical fire planning tool. It shows, for particular ranges of the independent variables, that H is dominated by U, which, in turn, is dominated by A—hence, that U and A overestimate the effective rate-of-spread.

METHODS

In typical fire planning applications, managers want to predict the rate-of-spread between two points in a spatially nonuniform environment, to project fire perimeters. The solution requires the Rothermel spread model to calculate spread values in spatial units chosen small enough, so that the environment is essentially constant within the unit. Proper characterization of the nonuniformity partitions the area into mutually disjoint homogeneous units, for example, the fuel cell honeycomb of Frandsen and Andrews (1979). Under the mosaic described by the partitioning, one can produce the rate-of-spread field using Rothermel's model, then apply the harmonic mean along the burn path to obtain the effective spread rate.

Derivation of H.—The rationale for using the harmonic mean derives from the mathematical definition of the rate of spread. Suppose a map showing a temporal sequence of fire perimeters is given. Assume that, along any given trajectory orthogonal to the fire perimeters, the Rothermel spread model successfully predicts the spread rate in mutually disjoint connected segments of the trajectory. The segments are chosen so that the variables of the rate-of-spread function are essentially constant over each interval; hence, a single rate-of-spread calculation characterizes each segment (Fig. 1). This characterization of the trajectory is the one-dimensional partitioning of the spread field into homogeneous units. Let l_i be the length of segment i. Define the effective spread rate, R_e, as the ratio of the length, L, that the flaming front travels between any two points on the trajectory, to the time, T, it takes to traverse L, burning continuously from one point to the other:

$$R_e = L/T. \tag{1}$$

The segments $\{l_i\}$ sum to L. The Rothermel model is used recursively to calculate the spread rate in each segment. Apart from the requirement for uniformity in each segment, the other limitations of the spread model apply (Rothermel 1982). If these limitations are acceptable, then it is possible to account for the effects of nonuniformity on the spread of the fire, whether the source of nonuniformity is fuel, weather, or topographic variations. The time that it takes the flame front to cross segment i, t_i, is

$$t_i = l_i/R_i \tag{2}$$

where R_i is the spread rate in segment i. If the transition times between segments are negligible,

$$\begin{aligned} T &= \sum_{i=1}^{n} t_i \\ &= \sum_{i=1}^{n} l_i/R_i. \end{aligned} \tag{3}$$

Substituting into (1), we get

$$R_e = L\Big/\left(\sum_{i=1}^{n} l_i/R_i\right) \tag{4}$$

or

$$1/R_e = \sum_{i=1}^{n} (l_i/L)/R_i. \tag{5}$$

Because the $\{l_i\}$ are positive and sum to L, we may form weighting coefficients $\{a_i\}$, where

$$a_i = l_i/L, \tag{6}$$

satisfying

$$0 < a_i \leq 1$$

and

$$\sum_{i=1}^{n} a_i = 1.$$

Using (6) in (5),

$$\begin{aligned} 1/R_e &= \sum_{i=1}^{n} a_i/R_i \\ &= 1/H. \end{aligned} \tag{7}$$

This is a generalized expression for the reciprocal harmonic mean—of spread rates, in this case. R_e is an H-type estimator that represents the effective spread rate between the end points of L orthogonal to the flaming front over time. The weighting coefficients are spatially weighted, corresponding to those used with A and U. The difference among the three estimators is only in how the weighting is applied.

Ordering of the Estimators.—In certain subdomains of the independent variables, the rate-of-spread function is convex (i.e., J- or U-shaped), or piecewise convex. This is inherent in the nonlinear and monotonic nature of the spread function,

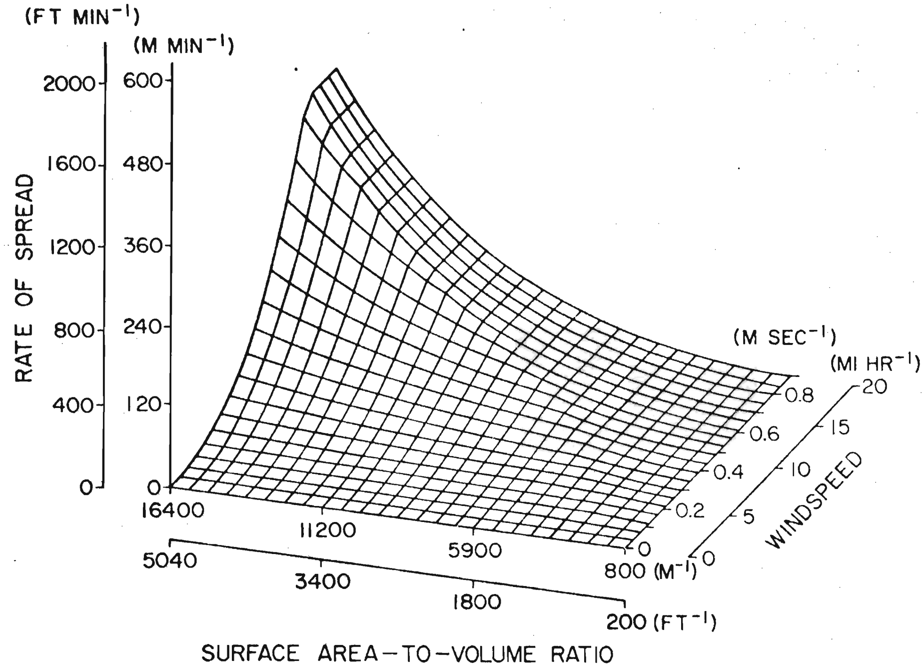

FIGURE 2. The rate-of-spread function for variable surface area-to-volume ratio, σ, and windspeed, u, with other independent variables held constant (NFDR Fuel Model L parameters and zero slope), is convex in particular subdomains of σ and u.

with respect to certain independent variables, for example, surface area-to-volume ratio and windspeed. Consider the graph of the rate-of-spread function for NFDR Model L grass (Deeming and others 1977) on flat ground, with moisture content at 5 percent (Fig. 2). In the ranges of the variables indicated, rate-of-spread is a positive function that exhibits rapidly changing values for high surface area-to-volume ratios, σ, and low to moderate windspeeds, u. For any given σ, the function reaches a plateau beyond a critical u. Similarly, for a given u, the function grows with σ until it reaches a critical σ, at which point it increases at a decelerated rate (Fig. 3). The result is that the rate-of-spread function is convex in particular subdomains; there, A, U, and H are well-ordered, independent of the weight distribution described by the $\{a_i\}$. Therefore, $H \leq U \leq A$, regardless of the particular combinations of σ and u that occur.

For example, the rate-of-spread function is convex in the interval 6,560–9,840 m^{-1} (2,000–3,000 ft^{-1}), all other variables held constant (Fig. 3). Suppose we have a burning environment described by the fixed conditions of the function in Figure 3, except that the fuel is a mixture of two grasses that differ only in σ. The grasses might be western perennial grass and western annual grass.[1] Let a proportion, p, of the burn path have a σ of 6,560 m^{-1} (2,000 ft^{-1}), and the remaining $(1 - p)$ a σ of 9,840 m^{-1} (3,000 ft^{-1}). Let $R(\sigma)$ represent the Rothermel rate-of-spread as a function of σ, with the other variables constant, as in Figure 3. Then

$$\begin{aligned} A &= pR(6{,}560) + (1 - p)R(9{,}840) \\ &= [R(6{,}560) - R(9{,}840)]p + R(9{,}840) \end{aligned} \tag{8}$$

[1] Personal communication with Jack D. Cohen, Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture, Riverside, CA 92507. May 1983.

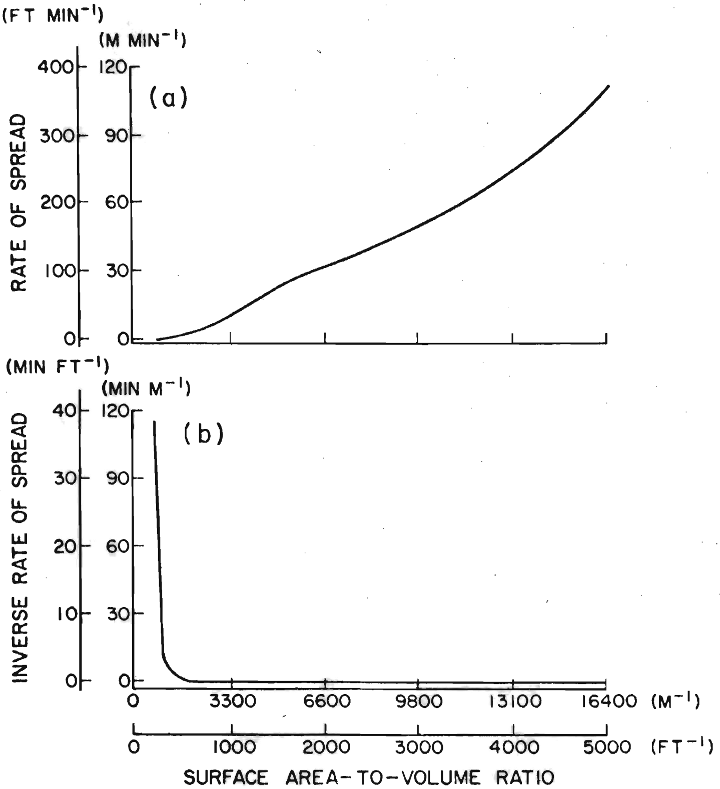

FIGURE 3. (a) Cross-section of rate-of-spread function in Figure 2 in the plane where windspeed is 0.45 m sec^{-1} (10 mi hr^{-1}), is piecewise convex; (b) inverse of function in (a) is also convex.

and

$$U = R[6{,}560p + 9{,}840(1 - p)] \tag{9}$$

and H is given by

$$1/H = p/R(6{,}560) + (1 - p)/R(9{,}840). \tag{10}$$

A plot of the estimators for p in the interval 0–1 clearly shows the ordering $H \leq U \leq A$, although the differences are small (Fig. 4). The three estimators converge at the end points of the interval because the fuelbed comprises a pure stand when p is 0 or 1; in these trivial cases of no nonuniformity, the estimators all give the same answer. Given that R is convex, A dominates U over the range of p, because—by definition of convexity—a chord drawn between any two points of a convex function will lie above the function. In this case, A represents the chord over $R(\sigma)$ for σ in the range 6,560–9,840 m^{-1} (2,000–3,000 ft^{-1}). Hence, whatever p is, the average surface area-to-volume ratio is between 6,560 and 9,840 m^{-1}, and $U \leq A$.

Jensen's inequality (Mood and others 1974, p. 72) formally explains why A dominates U: "Let X be a random variable with mean $E(X)$, and let g be a convex function; then

$$E[g(X)] \geq g[E(X)]." \tag{11}$$

In other words, the mean value of a convex function of a random variable dom-

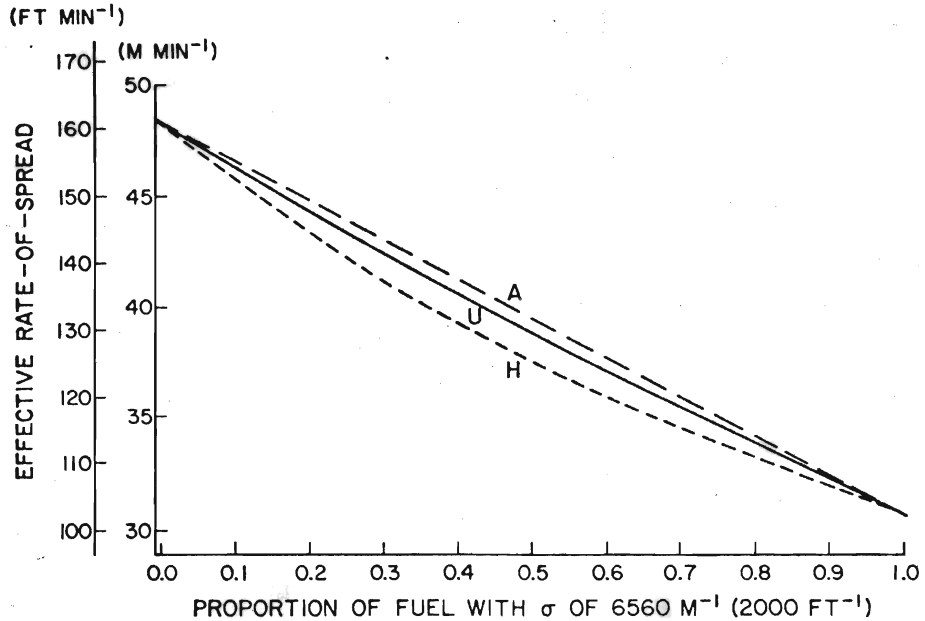

FIGURE 4. Ordering of effective rate-of-spread estimators obtained from the arithmetic mean of spread rates (A), spread using the average σ (U), and the harmonic mean of spread rates (H), over varying proportions of two NFDR Model L type fuels with different surface area-to-volume ratios. Burn path slope is zero.

inates the function value evaluated at the mean. This would explain the ordering of U and A observed by Frandsen and Andrews (1979). To explain the subordination of H, let

$$S(\sigma) = 1/R(\sigma). \tag{12}$$

Note that S is also convex (Fig. 3). Applying Jensen's inequality to S, we deduce that $1/H \geq 1/U$; because R is positive, this implies that $H \leq U$, and, by transitivity,

$$H \leq U \leq A. \tag{13}$$

A striking example of the disparity between the estimators follows.

Suppose that, in the previous example, the two fuels differ in surface area-to-volume ratio, in such a way that the average is 8,200 m^{-1} (2,500 ft^{-1}). Let a proportion p of fuel with σ of 4,920 m^{-1} (1,500 ft^{-1}) be mixed with $(1 - p)$ of fuel with

$$\sigma_2 = (8{,}200 - 4{,}920p)/(1 - p).$$

Therefore, when p is 0.5, σ_2 is approximately 11,500 m^{-1} (3,500 ft^{-1}). Estimates calculated for p in the interval 0–0.5 confirm (13) and illustrate some other noteworthy features (Fig. 5). Clearly, U cannot reflect the dynamics of changing proportions of the hypothetical communities. A and H take divergent paths with increasing p: A growing larger, H smaller. The trend of H is more plausible, because increasing p mixes in more slow-burning fuel (remarkably, H, in spite of its imposing nonlinearity, decreases almost linearly with p). The nonlinear response of A reflects the rapidly increasing influence of $R(\sigma_2)$ and exhibits a flaw in this estimator.

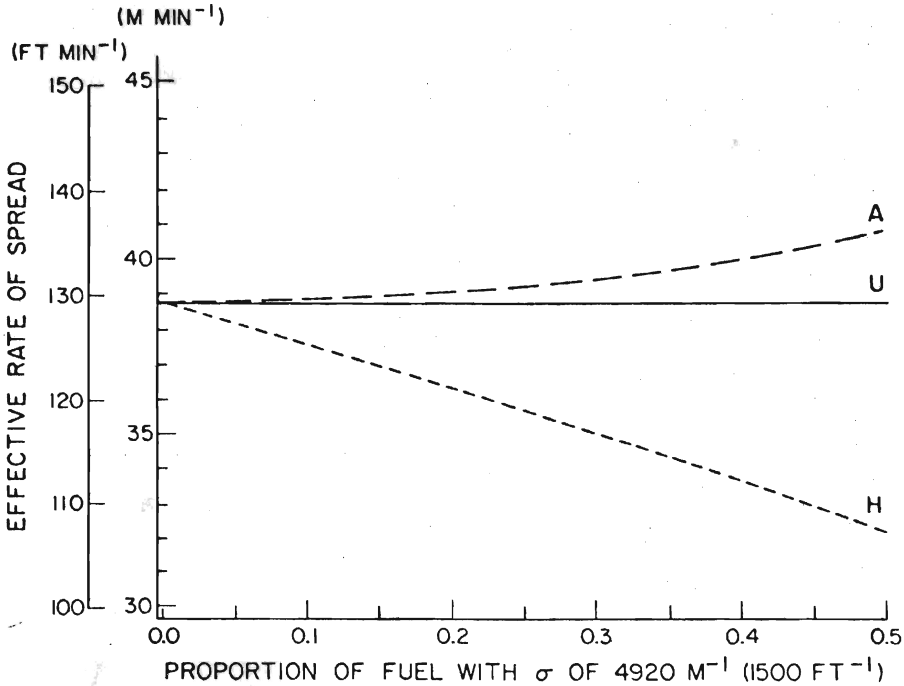

FIGURE 5. Effective rate-of-spread estimators from the arithmetic mean of spread rates (A), spread using the average σ (U), and the harmonic mean of spread rates (H), diverge as p increases. The fuel is a mixture of two NFDR Model L type fuels differing in σ, proportion p with σ 4,920 m^{-1} (1,500 ft^{-1}), $(1 - p)$ with σ such that the average σ is 8,200 m^{-1} (2,500 ft^{-1}).

Consider an example in which a fire spread at the rate of 10 m min^{-1} over 900 m of the burn path, and 100 m min^{-1} over 100 m:

$$A = 10(900/1{,}000) + 100(100/1{,}000) \\ = 19 \text{ m min}^{-1}$$

and

$$H = 1/(0.9/10 + 0.1/100) \\ = 11 \text{ m min}^{-1}.$$

Using A in (1), the elapsed time that the fire would take to cover 1,000 m would be 53 min. H yields an elapsed time of 91 min. The true elapsed time can be found from (3):

$$T = 900/10 + 100/100 \\ = 91 \text{ min.}$$

The same elapsed times obtained from H and (3) is expected, considering that H incorporates the time-distance relationship. In fact, the flaw in A is that it weights the spread rates spatially, not temporally; A would indeed be appropriate for the effective spread rate, if it used weights $\{b_i\}$ that express the proportion of the time the fire burns at the rates $\{R_i\}$, over L. Let A_d represent the temporally-weighted arithmetic mean:

$$A_d = \sum_{i=1}^{n} b_i R_i$$

where

$$b_i = t_i/T.$$

The $\{t_i\}$ are positive and sum to T, so the $\{b_i\}$, like the $\{a_i\}$, are constrained by

$$0 < b_i \leq 1$$

and

$$\sum_{i=1}^{n} b_i = 1.$$

From (2),

$$\begin{aligned} A_d &= \sum_{i=1}^{n} (l_i/R_i)R_i/T \\ &= \sum_{i=1}^{n} l_i/T \\ &= L/T \\ &= H. \end{aligned} \qquad (14)$$

The time-distance relationship provides a duality of weighting coefficients: those spatially weighted are appropriate for H, and those temporally weighted, for A. Because A_d equals H, the choice of which to use depends on whether it is more convenient to describe the nonuniformity of the rate-of-spread field in terms of spatial or temporal variability (spatial descriptions are more common). Nonuniformity due to weather is most conveniently described spatially (Fujioka 1983), but weather variables are also highly transitory, more so than any of the factors that affect fire spread in the Rothermel model. Therefore, both spatial and temporal variability must be considered when analyzing the effective spread rate over extended periods.

Conclusions

The Rothermel rate-of-spread model accounts for fuel, weather, and topographic influences in the propagation of the flaming front, provided that those variables are uniform in some unit of space. To deal with spatial nonuniformity in any combination of these variables and, therefore, in the rate-of-spread field, one must first partition the field into homogeneous units. This partitioning allows the use of the spread model within each unit. The spatially weighted coefficients follow from the partitioning process. The weighting that results resembles a discrete probability distribution of the spread rates. The degree of nonuniformity that exists in any given situation is reflected in the range of this distribution.

The spread distributions contain a nontrivial twofold directional dependence. First, the orientation of the trajectory determines the spatially weighted coefficients along its length. Secondly, rate-of-spread is itself direction-dependent, being a function of wind and topographic slope.[2] Spread rate distributions might differ drastically between trajectories through the head and the flank of a fire. In such cases, multiple sets of spread distributions might be necessary to describe fire spread along several trajectories. However, a single set of coefficients might still be appropriate in certain instances—e.g., when it is not feasible to characterize

[2] Albini, Frank A. Memorandum to R. C. Rothermel: "Combining wind and slope effects on spread rate." Northern Forest Fire Laboratory, Missoula, MT 59806. Jan. 19, 1976.

the anisotropy of the spread field, or when spread estimates are desired for only a single aspect of nonuniformity, such as the distribution of fuel types on a burn path.

After the spread distribution along the trajectory is obtained, the effective spread rate can be determined from the harmonic mean, with spatially weighted coefficients. If the assumptions used to derive (7) are correct, U and A overestimate the effective spread rate. This does not imply that U and A estimators are never useful. An A-type estimator, A_d, can be used to determine the effective rate-of-spread, when the weighting coefficients are temporally weighted, as shown in Equation (14). In characterizing nonuniformity, we ultimately use U within each cell, because we resort to a discretization of what is really a continuously varying field. The point, then, is that each estimator fits the application in a particular context. Therefore, while the estimators are useful tools with which to handle nonuniformity, the context of their use must be clearly defined.

This investigation concentrated on a relatively limited domain of the rate-of-spread function. Its focus has been the rate-of-spread of cured fine fuels, with emphasis on the effects of surface area-to-volume ratio. The analysis needs to be broadened to a variety of fuel-weather-topography complexes.

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Biological Nitrogen Fixation in Forest Ecosystems: Foundations and Applications

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Biological nitrogen fixation is a powerful tool available to foresters for managing stand nutrition, yet its commercial use is almost nonexistent in North America. Part of the reason